

A DENSITY ANALYSIS OF A DEVELOPING  
DEEP TROUGH IN THE WESTERLIES

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D. C. Bayly







A DENSITY ANALYSIS OF A DEVELOPING  
DEEP TROUGH IN THE WESTERLIES

by  
Donald C. Bayly  
Lieutenant, United States Navy

Submitted in partial fulfillment  
of the requirements  
for the degree of  
MASTER OF SCIENCE  
IN AEROLOGY

United States Naval Postgraduate School  
Monterey, California  
1952



This work is accepted as fulfilling  
the thesis requirements for the degree of

MASTER OF SCIENCE  
IN AEROLOGY

from the  
United States Naval Postgraduate School



## PREFACE

This paper presents an investigation of a rapidly deepening trough in the westerly zonal circulation over the United States. Material was gathered during a continuous five day period at intervals of 24 hours in a dense radiosonde network. The analysis was conducted in 2 kilometer layers from the surface to 20 kilometers with density as the primary synoptic variable.

The purposes of this investigation were first, to present in detail the changes in the density field over the United States during the development of an unusually deep trough; second, to determine the primary factors causing the development and the ensuing effects of this development upon the pressure pattern at the ground; and lastly, to investigate as many dynamic and synoptic features of the system as possible in order to further the understanding of all factors influencing the weather.

The author is indebted to Associate Professor Frank L. Martin of the Aerology Department, and wishes to express gratitude for the advice, suggestions and guidance given during the investigation.



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# TABLE OF SYMBOLS AND ABBREVIATIONS

$C_p$	Specific heat of dry air at constant pressure
$g$	Acceluation of gravity
GCT	Greenwich civil time
$p$	Pressure
$\rho$	Density
mb	Millibar
$t$	Time
$R_d$	Gas constant per gram of dry air
$T$	Temperature
$K$	$R_d/C_p$
$w$	Vertical component of wind velocity
$\theta$	Potential temperature
$\nabla$	Vector del operator
$\nabla_H$	Horizontal gradient
$\mathbf{v}_H$	Horizontal wind velocity vector
$\Delta z$	Thickness between two pressure surfaces
$\delta p$	Pressure change in the vertical between two fixed horizontal levels
$\Delta p$	24-hour change of pressure at a fixed elevation
$(\frac{\partial \rho}{\partial t})_{24}$	Local rate of change of density, per 24 hour
$(\frac{\partial p}{\partial t})_{24}$	Local rate of change of pressure, per 24 hour
After Fleagle [4]	
D	Density term, positive for increasing density
P	Local compression term, positive for compression



A        Advection term, positive for cold advection  
V        Vertical motion term, positive for upward motion  
HC       Horizontal mass convergence term, positive for convergence  
VC       Vertical mass convergence, positive for convergence  
 $\Delta p_s$    Surface 24-hour pressure tendency  
 $\Delta p_8$     8 kilometer 24-hour pressure tendency





## I. INTRODUCTION

Until recently, a network of radiosonde stations dense enough to completely encompass a typical wave in the westerlies or a typical synoptic situation to great height, has been lacking. As a result, investigations along synoptic lines have tended to lag theoretical considerations. For this reason the author has selected actual synoptic data for an investigation. Density was chosen as the primary variable because it is known that density most nearly reflects the combined effects of all the other variables entering into the mechanism of wave formation and dissipation and the resultant weather.

Since the first statistical upper air relationships between density, temperature and pressure were found, meteorologists have attempted the systematic investigation of weather on the basis of these factors and have made prognostications based upon rules, theories and facts gleaned from such research. Early investigators reach the primary conclusion that density changes were continually taking place, not only at the surface, but also at considerable height. It was found that large density changes in the lower troposphere tend to be compensated and in many instances over-compensated by large scale density changes of opposite sign in the lower stratosphere. This balance or over-balance prevents extremely large pressure variations at the surface.



Rossby [9] attempted to explain this balance in terms of adiabatic compression and warming of the column of air beneath a large and dense layer of air aloft. This suggestion led to a reasonable explanation of a layer at about 8 kilometers where the density remained practically constant. This layer was also proven to be the area of maximum pressure change. Rossby however, assumed no horizontal velocity divergence, which later proved to be a poor assumption.

Other investigators then showed that many factors contribute to, and influence the changes in density at various layers of the stratosphere and troposphere. Bjerknes [1], using the equation of continuity and the gradient wind equation, interpreted the distribution of horizontal mass divergence. Bjerknes and Holmboe showed, with a defined super- or sub-critical wind speed, the horizontal mass convergence and divergence at various positions with reference to troughs and ridges. Durst and Sutcliffe [2] proved that the vertical components in the equations of motion, which had previously been ignored, also contributed to the mass divergence and density changes. Rossby, Haurwitz, Holmboe [9, 5, 6], and others have since investigated and offered models for frontal waves, troughs and ridges in terms of continually more representative equations of motion. Recently, reliable methods for evaluation of vertical motions have been contributed by Panofsky [8].

This work is an attempt to analyze the mechanisms involved in a developing long wave in the westerlies. The analysis has been furnished by means of local density changes over a vast network of stations.



These local density changes, as shown above, are a result of many factors, some of which can and will be estimated, by means described later. An analysis of the developing trough and ensuing closed low has been drawn by means of mean density change charts for layers 2 kilometers in thickness from the ground to 20 kilometers. A second investigation has been made by cross sectional diagrams along a mean route of movement of density change centers. Time limitations prevented construction of similar diagrams along other instructive routes. The author has finally made qualitative remarks concerning interesting features noted as well as limited quantitative comparisons with results of other investigations. Answers to the following questions were particularly sought.

1. What are the levels of maximum density change?
2. Are there specific regions of density rise and fall along the vertical, and if so, which has the greatest effect on the pressure tendency at the ground?
3. What is the level of no density change?
4. What significance, if any, is to be attached to rise and fall centers of density occurring across this layer?
5. What are the movements of the  $\frac{\partial \rho}{\partial t}$  centers at various layers, and what is their possible use as forecasting tools?
6. What are the tendencies of  $\frac{\partial \rho}{\partial t}$  centers?
7. What are the relations of  $\frac{\partial \rho}{\partial t}$  centers in the horizontal to the position of long wave and short wave troughs?





## II. PROCEDURE AND TECHNIQUE

It was felt that the easiest means of obtaining day to day local density was to plot the station radiosonde soundings on a T- $\theta$  diagram picking the density at the mid-point of each 2 kilometer layer directly off the diagram with the aid of the station pressure height curve. The densities at 2 kilometer levels were then plotted on a day to day density height curve. Daily density curves were drawn rather than using the relation between pressure and density expressed in the hydrostatic equation

$$\frac{\partial}{\partial t} (\Delta p) = -g \Delta z \frac{\partial \rho}{\partial t} \quad (1)$$

because it was felt that in many cases station soundings for succeeding days would not be complete nor would always extend to heights desired. By means of the day to day density curves however, many incomplete soundings could be extrapolated to desired heights, or missing portions of soundings could be filled in. Data for the soundings was taken from the "Daily Upper Air Bulletin", and frequently was partially incomplete. These density curves also very clearly indicated the general areas of maximum  $\frac{\partial \rho}{\partial t}$  change as well as the mean region of  $\frac{\partial \rho}{\partial t} = 0$ .

The 24 hour change in local density was easily taken from the density height curves at the mid-point of each 2 kilometer layer. A 24 hour basis was selected to eliminate the diurnal change. From this





data, layer charts were constructed representing the mean change of for 24 hour periods. Isolines were drawn for each  $.001 \frac{\text{kg}}{\text{m}^3}$  of change, and troughs and ridges at the surface and 500 mb levels were superimposed.

Centers of density change were tracked at each level and a track chosen for a cross section which most closely represented the mean track of the movements of centers during this period. This mean route was selected in order that any relationship between successive centers in the horizontal, as well as any relationship in the vertical, would appear. Other cross sections would no doubt prove instructive, but time limitations prevented such further investigation.



### III. SYNOPTIC SITUATION

On 01-0030 GCT November 1950 the subtropical Eastern Pacific high was situated midway between Hawaii and the coast of California (See Plate I). A Pacific polar front was situated along parallel of latitude  $40^{\circ}$  N, with a series of two unstable waves along the front to westward, and an occluded wave approaching the California coast. A great basin high of 1028 mbs was stationary in the area of Nevada and Colorado. A cold polar high of the same central pressure was moving slowly southward in the rear of a cold front which had previously been the typical stationary front. A well defined cold front stretched across the United States in a southwest-northeast direction, ending in a 996 low and occlusion over the southern section of Hudsons Bay. This front was moving slowly eastward showing large wind shear and several small waves along it. One of these developed to a central value of 999 mbs on November 02-0030Z in the Great Lakes area, and moved rapidly NE out over the Atlantic (See Plate II).

On succeeding days the occluded wave from the Pacific moved inland over the western states by 03-0030 GCT to the vicinity of Texas with a large high of 1035 mbs, which was situated over the Continental Divide area (See Plate III). An unstable wave formed at the junction point of the occlusion and the now nearly stationary cold front, namely, in SW Texas. This is the cyclone which was of primary interest in this investigation. The polar high moved southward in the lee of the Rockies



behind this cold front and reached the Gulf of Mexico in the vicinity of Brownsville, Texas, on 05-0030 GCT (See Plate V). By 04-0030 GCT the closed low which had developed on the stationary section of the cold front had deepened to 996 mbs and moved north eastward, buffeting the New York City area with gale force winds on 05-0030 GCT (See Plate IV and V).

The 500 mb level for the period 1-2 November indicated moderate high index conditions with but one small amplitude long wave trough oriented north-south, and situated over the midwestern states. A ridge of similar orientation was situated over the Great Lakes region (See Plate I). During the period 1-3 November, this long wave trough, which will be called the primary trough, moved slowly eastward showing stable characteristics. It is perhaps significant that a small short wave trough appeared over the northern coast of British Columbia on 02-0300 GCT, moving more rapidly than the primary trough. This short wave trough was well indicated at the 500 mb level. By 03-0300 GCT the short wave trough reached a position just west of the Great Lakes region while the primary trough was situated about  $10^{\circ}$  longitude to the eastward. This gave the contours of the 500 mb level the appearance of a very broad, shallow trough over the central United States. On 04-0300 GCT the now unstable primary trough at 500 mbs was evident. It had moved eastward very little, but had deepened considerably (See Plate IV). It extended along the 90th meridian from the Great Lakes to the Gulf of Mexico. On 5 November this unstable wave moved slowly







eastward reaching the Atlantic by 06-0300 GCT. A closed low vortex over the Chicago area became evident in this trough just at the 300 mb level on 04-0300 GCT and appeared slightly to eastward, over Buffalo on 05-0300 at the 500 mb level. At this time the closed low at the surface was situated south eastward of the Buffalo area (See Plate V).



#### IV. FEATURES OF LAYER CHARTS

Significant features of these charts must be subdivided into three general layers, as each layer presented a different picture. General layers to be discussed in order are: first, 2 kilometer layers included between the ground and 8 kilometers; second, the layer including 8 to 10 kilometers; and third, the layer 10 to 20 kilometers.

On the 1-2 November charts of the lower level (1-8 kilometers) (pg 1) the significant features were two large rise centers immediately behind the cold front at the surface, and extending to higher levels with axes slightly tilted to the northwest. In levels to 2 kilometers these rise centers were extended in the direction of the high pressure centers. During the period 2-3 November, the rise centers continued to move behind the surface cold front and slightly in advance of the polar high. A fall center moved into the region in the lee of the Rockies formerly occupied by a rise center. The axis of the fall center also tilted northwestward. On the 3-4 charts the rise centers behind the cold front moved farther apart, one moving to the northeast, and the other southward over the Gulf of Mexico. At this time, the surface high split, and the two highs followed the same routes as their associated rise center. The area between the two rise centers changed from slight rise in density on the 2-3 November charts to a fall center by the 4-5 November charts. In this area at the surface, the front became stationary. It was noted that at the 500 mb level (4-6 kilometers), troughs were definitely associated



with each rise center. Theoretically the trough should appear midway between fall and rise centers. This was the case at ground level. However, at higher layers up to 500 mbs the trough appeared displaced toward the rise centers with small rises ahead. This is in agreement with Figure 10 of [4] by Fleagle. Ridges were positioned slightly in advance of fall centers at the same level. No attempt was made to trace troughs or ridges above 500 mbs.

Change centers of the level 8-10 kilometers were small during the period 1-2 November, some having the sign associated with levels below 8 kilometers and others having the opposite connected with layers above 10 kilometers. As previously mentioned, this level (8-10 kilometers) was found to be the general layer of no density change. However, a large fall center appeared at this level on the 2-3 November chart in the region in the lee of the Rockies. On succeeding days this fall center, whose vertical axis extended into a fall center above 10 kilometers, moved southeastward over the southeastern states, where it remained practically stationary over the period 4-5 November (See Plates VI, VII and VIII). A rise center replaced the fall center during succeeding 24 hour periods.

The general features of the layer 10-20 kilometers consisted, during the period 1-2 November, of a large fall center in the lee of the Rockies, and a rise center extending over the area from the continental divide to the Pacific coast. The axis of this center tilted slightly





to the NW. The fall center was located during the period 1-2 November directly over the rise center of the level 4-6 kilometers of the same period. On the 2-3 November charts the fall center had moved SE over the midwestern states. On succeeding days the fall center split exactly as had the rise center at lower levels described above. One center moved NE after the split. The other moved over the southeastern states where it remained stationary over the period 4-5 November. After the split, the vertical axes of the two fall centers were also tilted slightly to the NW.

Features noted at all levels follow. Tracking of centers at levels below 8 kilometers was accomplished by estimating trajectories and using 90 percent of the observed winds. It was found that centers in the layer 6-14 kilometers moved faster than those above or below this layer. As a result of this relative motion, quantitative superposition of density change varied from period to period. This is more evident on the vertical cross section, described in Chapter V. A very dominant feature noted at all levels during the period 1-3 November, but particularly at the 10-14 kilometer levels, was the orientation of rise or fall centers and surrounding isolines along the eastern slope of the Rockies. This feature was to be expected at low levels, but appeared unusual at the elevations found. Such orientation was noticeable to 16 kilometers, and appeared most evident during the period when the wind flow at the 500 mb level was NW. During this period, increased central values of fall were noted, and could possibly be explained as





resulting from increased downward velocities (See Plates VI, VII and VIII). A final feature noted was the extension of centers of fall during the period 3-5 November. Above 8 kilometers, the fall centers had moved to the region over Mississippi by 4 November according to regular tracking principles. At this time a new fall center aloft suddenly appeared, simultaneous with the appearance of the 500 mb vortex, in the form of an extension from the fall center which had been tracked to Mississippi. At levels below 8 kilometers, along the line of the extension, the sign of  $\frac{\partial p}{\partial t}$  was positive (See Plate VIII). The author speculates that these extensions, which were evident on days following appearance of a closed vortex at 500 mbs, could be explained as changes in density produced by vertical motion, upward through the vortex from the surface to 8 kilometers, and downward above that level. This is in agreement with principles given by Hsieh, Yi-Ping [7] . Observations made concerning possible vertical motions in a vortex are not offered as proof, but merely as interesting features offering possible further investigation along methods indicated in [8] .



## V. INVESTIGATION BY CROSS SECTION

Figures 5-9 display a cross section along a route approximately that of the largest  $\frac{\partial p}{\partial t}$  centers. There is evidence that there exist two regions of maximum absolute values of  $\frac{\partial p}{\partial t}$ ; surface to 3 kilometers, and 12-14 kilometers. Also evident is the reversal of sign at approximately 8 kilometers in the mean. Although the axis between centers of the same sign at the two levels appears to remain at nearly the same angle, it was noted by tracking, that the centers aloft in general moved more rapidly than at the lower levels. There is evidence as well, that as centers reached the lee of mountains, they moved downward slightly. As a consequence there resulted a change of density across the layer previously noted as an area of approximately no density change. Here again is evidence of possible density change resulting from adiabatic warming as a consequence of impressed downward velocities in the lee of mountains. (See Figures 5 and 6). Additional evidence exists for upward velocities and consequent increase of density across the mean layer of no density change. (See Figures 5 and 6). The region noted was that between stations #398 and 879 in the Pacific coast upslope area of the Rockies during the period when possible impressed downward velocities were occurring in the lee.

Local pressure tendencies for 24 hour periods are indicated below station numbers in Figures 5-8. It is evident quantitatively, that surface pressure tendencies are determined by superposition of rise and





fall values over the station, the numerically larger value determining the sign of the surface tendencies. In several cases, although ground densities are rising, the surface pressure tendency carries the sign of the fall center aloft. Figure 8 also displays two cases where maximum falling surface pressure tendencies are correlated directly with fall centers at all levels.

Trough lines as long dashes, are indicated as actually observed at levels to 500 mbs, as is the tropopause. It was noted that the tropopause descended to 8 kilometers in the two cases.

As has been previously stated, the level of no density change fluctuated slightly between 8 and 10 kilometers. Density change centers of small magnitude crossed this level in the region above mountains, giving several levels of  $\frac{\partial \rho}{\partial t} = 0$ . Note station #879 of Figure 5. A 24-hour pressure fall of -25 mb appears at this station at an elevation of 20 kilometers. Next day this pressure-change center has moved downward to 14 kilometers. If this is typical it would indicate the mechanism of the "triggering" action of the Rockies in intensifying troughs. Note also that in the instance cited the higher level of  $\frac{\partial \rho}{\partial t} = 0$  appears to be about 20 kilometers or higher.

Comparison of results of this investigation and those given in [4] are made in Figures 10-13. The reader is referred to that paper for details. Generally, Fleagle has, with the aid of the following formulas applied to 132 actual synoptic situations, determined quantitatively the





the specific percentage contributions in the various 100 mb layers to density change.

$$D = \frac{1}{p} \frac{\partial p}{\partial t} \quad (2)$$

$$P = \frac{(1-k)}{p} \frac{\partial p}{\partial t} \quad (3)$$

$$A = \frac{\nabla_H \cdot \nabla_H \theta}{\theta} \quad (4)$$

$$V = \frac{\omega}{\theta} \frac{\partial \theta}{\partial z} - \frac{1}{\theta} \frac{d\theta}{dt} \quad (5)$$

In order to compare results, Fleagle's mean data for percent density change has been reduced to mean density within 2 kilometer layers by means of the standard atmosphere. It is believed that errors resulting would be slight as Fleagle's data is a mean of many synoptic situations. A further assumption was necessary as Fleagle's data is based on 12 hour tendencies. The author has assumed that rate of change of  $\frac{\partial p}{\partial t}$  was constant for 24 hours. This assumption is reasonable as the gradient of  $\frac{\partial p}{\partial t}$  was fairly uniform at specific levels. In addition, all density change centers were converted to pressure change centers by means of Equation 1.

In [4] Fleagle classified data into 6 specific situations as follows:



Ra	$\Delta p_s > 0$ , $\Delta p_o > \Delta p_s$
Rb	$\Delta p_s > 0$ , $\Delta p_o > 0$ , $\Delta p_o < \Delta p_s$
Rc	$\Delta p_s > 0$ , $\Delta p_o < 0$
Fa	$\Delta p_s < 0$ , $\Delta p_o < \Delta p_s$
Fb	$\Delta p_s < 0$ , $\Delta p_o < 0$ , $\Delta p_o > \Delta p_s$
Fc	$\Delta p_s < 0$ , $\Delta p_o > 0$

Data from this investigation was classified accordingly and superimposed on Fleagle's adjusted curves. In many instances the similarity is evident. In particular, the similarity of shape and the point of no pressure change were noted. Tropopause levels also are in agreement in most cases. As can be expected, the curves do not agree quantitatively. This is primarily because the surface pressure change characteristics of the individual cases do not conform exactly to those of the mean cases studied by Fleagle. Furthermore, the conversion to 2 kilometer layers by means of the standard atmosphere, as well as the assumption of little change in density gradient, may introduce additional differences. Definite disagreement, however, is noted in the curves of stations near the Rockies. Levels of maximum pressure change are also in disagreement here (See Figures 5-6).



Figure 10 of [4] represented by Fleagle a vertical cross section showing average spatial distribution of  $\frac{1}{p} \frac{\partial p}{\partial t}$  relative to trough and wedge lines. Considerable agreement between cross sections of this particular investigation and Figure 10 of [4] may be noted. It should be mentioned, however, that Figure 10 as well as Figures 1-6 of 4 represent mean percentage change per 100 mb layer. As a consequence, though Fleagle's values of percent density change (before adjustment) in the low stratosphere are larger than those in the low troposphere, actually in terms of 2 kilometer layers of absolute pressure change, those in the low troposphere are larger. Results of this specific investigation agree well with Fleagle's figures although no attempt has been made to extrapolate trough lines to elevation above 500 mbs.

Pressure tendencies placed above each curve of Figures 1-8 indicate the difference between summation of local pressure changes at all levels over a particular station and the observed 24 hour ground pressure tendency. This gives the pressure change that must be attributed to elevations above 20 kilometers. This difference averaged 2 mbs. Striking departure from this average is noted at station #879, Edmonton, Alberta, in the lee of mountains. Values indicated in Figures 2 and 3 were extremely large, 25 mbs, and were checked and rechecked. The exact value but of opposite sign on succeeding 24 hour periods lends support





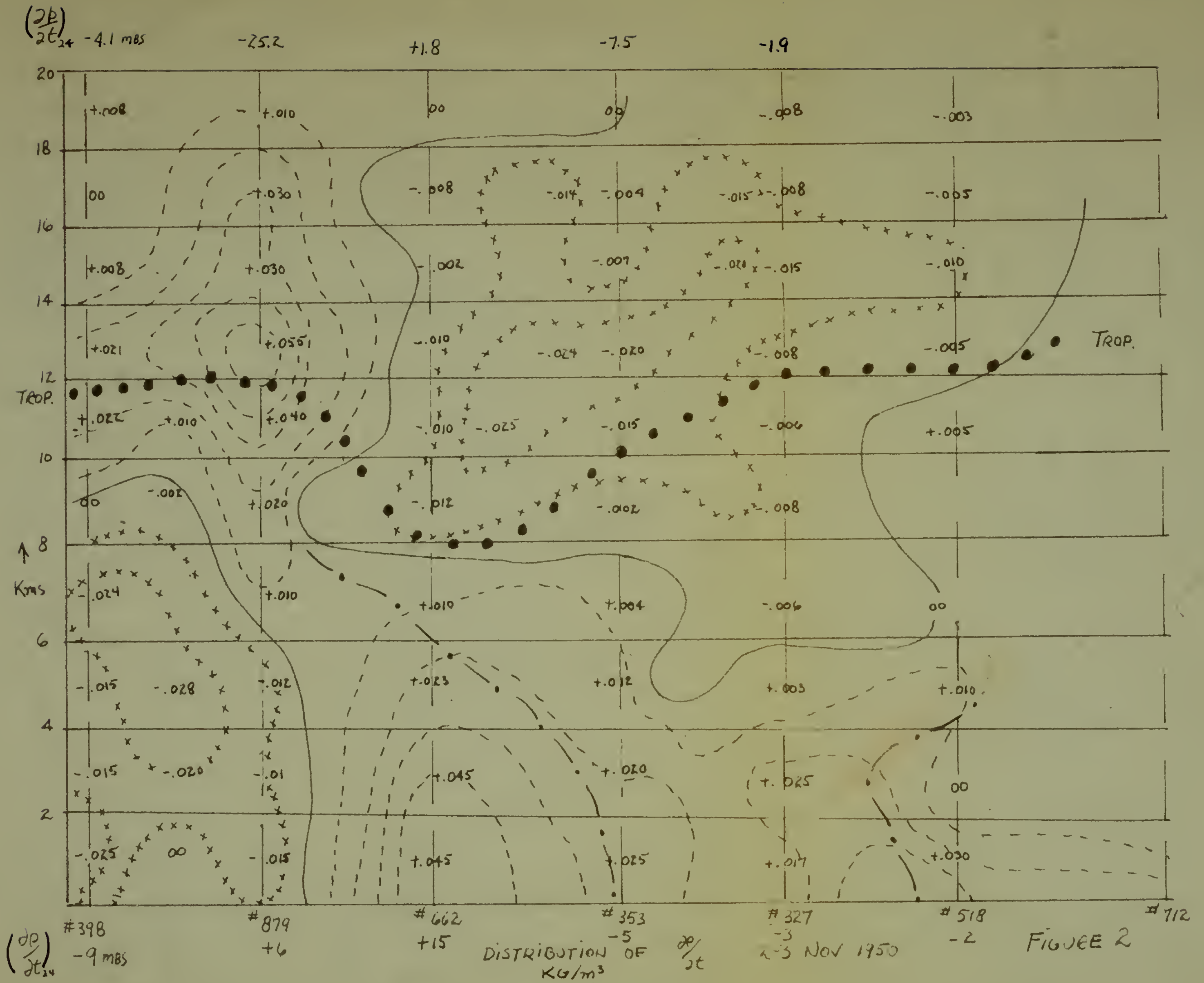
to their actual existence. The appearance of these large values during periods where previous evidence of large scale vertical motions was evident, lend support to that theory. This leads to the assumption that during the period 1-3 November, there were possibly three levels of maximum density change over the continental divide area.



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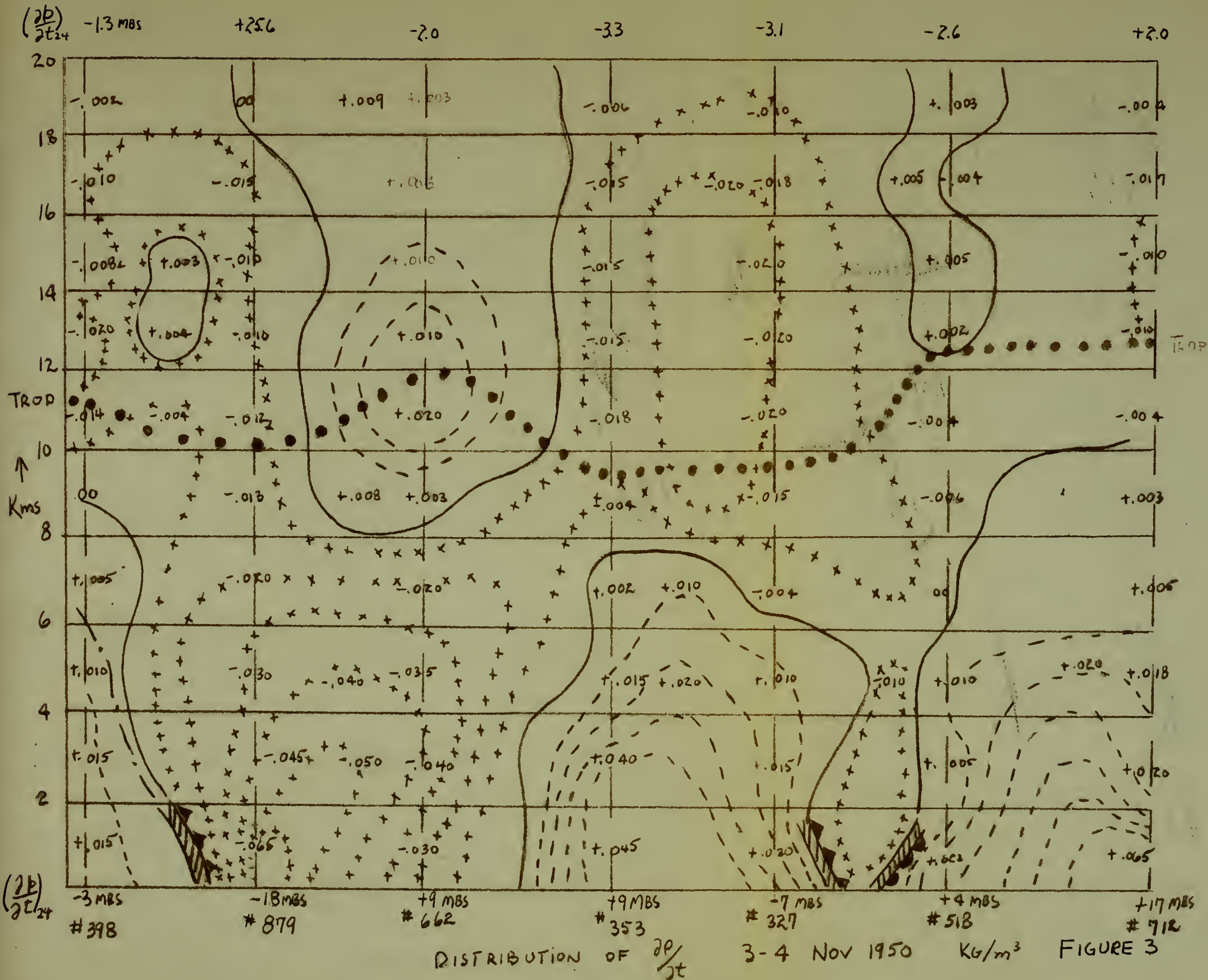
















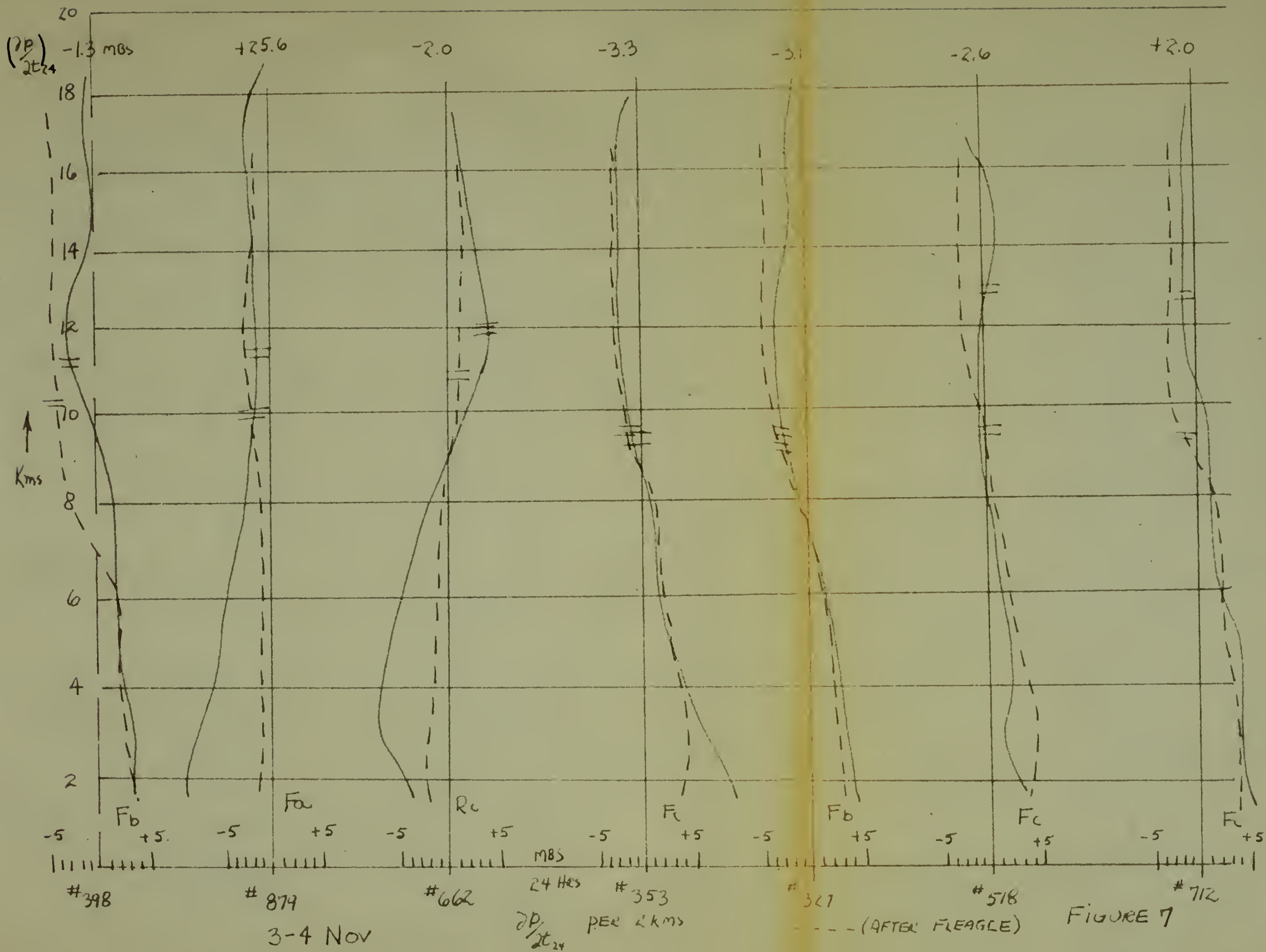


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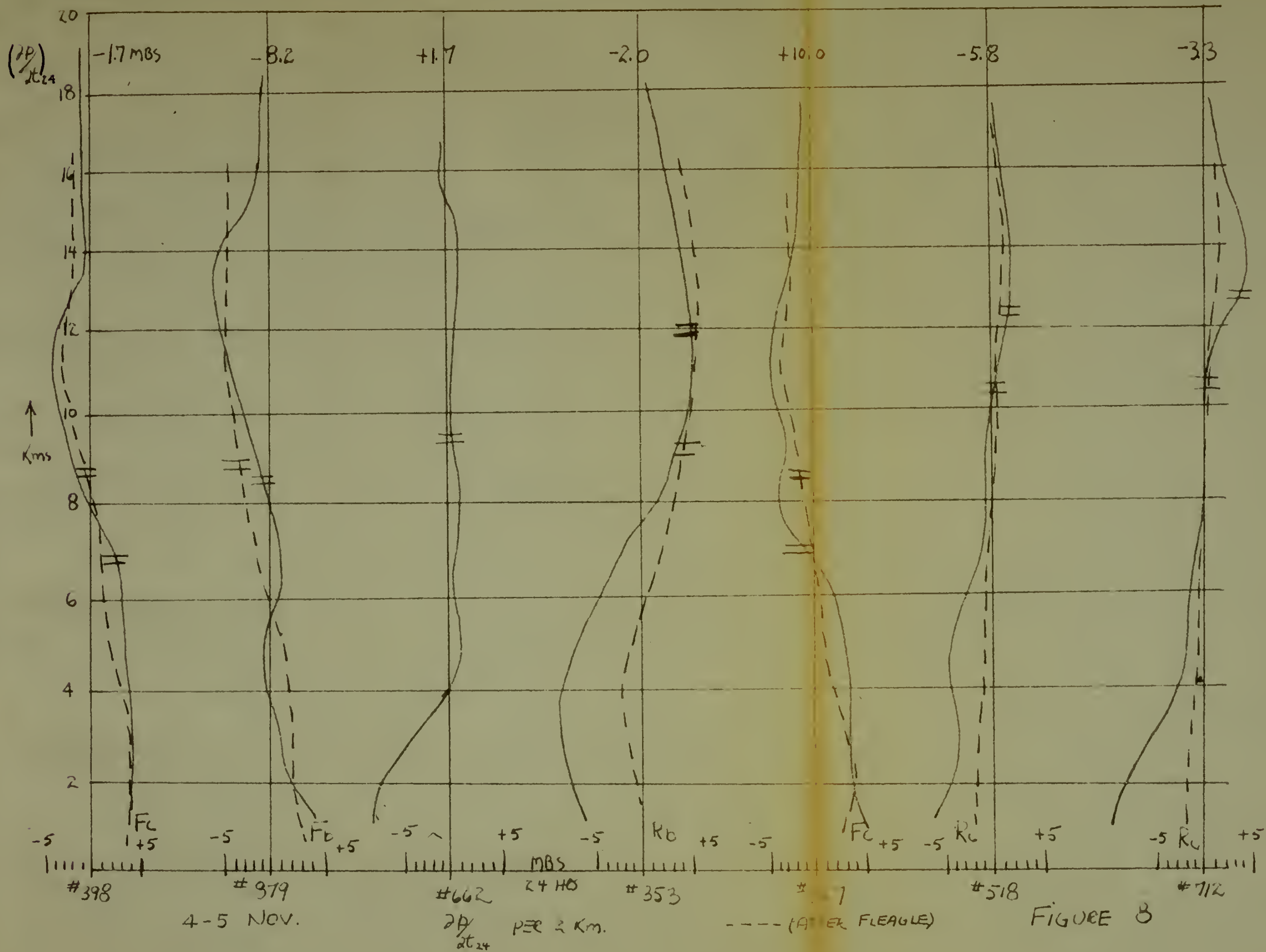














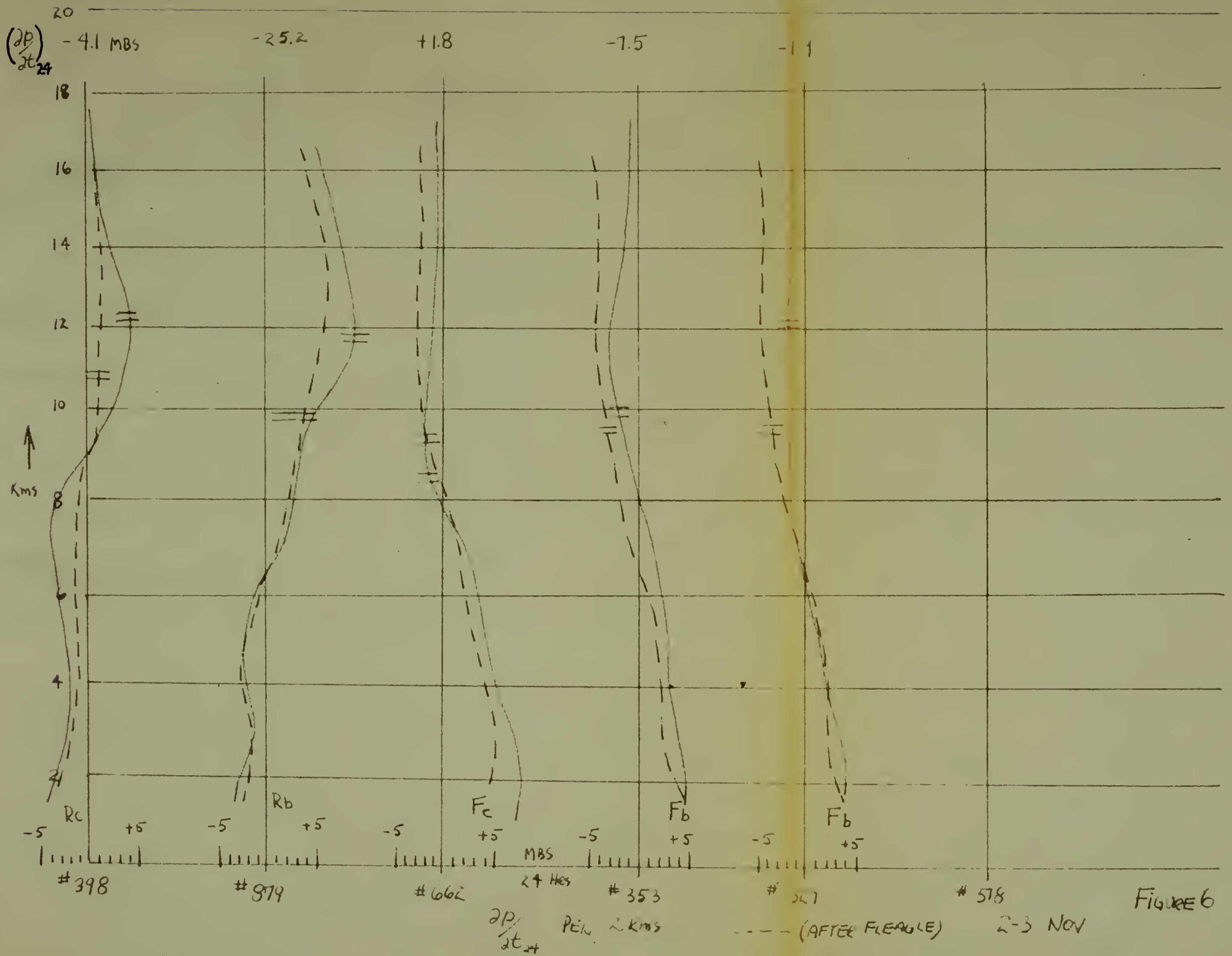


FIGURE 6



## VI. RESULTS AND CONCLUSIONS

Investigation of density change by methods described in this paper are laborious, but give results of great detail. Specific facts noted were as follows. Two regions contribute mostly to ground pressure tendencies, 1-4 kilometers and 12-16 kilometers, with density changes in the lower troposphere of greater magnitude during the period of this investigation. There is however, evidence of possibly a third level of maximum change of  $\frac{\partial \rho}{\partial t}$ , above 20 kilometers.

On the area immediately ahead of troughs at the ground, the associated fall center aloft determines the sign of surface pressure tendency, while immediately to the rear of the trough, the associated density rise center determines the sign of the ground pressure tendency. Actually, the general shape, size (in the horizontal and vertical), and speed of movement of centers at two levels determines the sign and amount of ground pressure tendencies. This has been noted previously. This superposition of centers is particularly evident in the developing of the specific cyclone investigated. During periods of maximum ground pressure fall, superposition of fall center over rise centers did not exist, but rather, falls were noted from the ground to 20 kilometers (See Figure 4.).

In general, a layer of no density change was located in the region 8-10 kilometers, and as found more accurately by means of the station mean density curves, was located at 8.5-9 kilometers. As previously noted, small centers of density change approximately one third the general values of levels above, did cross the layer of no density change.







Instances of this nature were noted only in the lee of mountains. (See Figure 1.). This caused the level of  $\frac{\partial \rho}{\partial t} = 0$  to become multiple, one below 6 kilometers and one above 9 kilometers. No specific heights were evident in such cases. These added changes in density in areas of generally slight density change enlarged the area of falling pressure in the upper troposphere and the lower stratosphere, increasing its vertical extent about 2 kilometers, and along the route of vertical cross section, often lowered the level of  $\frac{\partial \rho}{\partial t} = 0$  to 6 kilometers. (See Figures 1-5). The author believes this factor was primary in producing the rapid deepening of the investigated ground frontal cyclone. The vertical axis between centers of like sign appeared to vary only slightly, but tracking of centers at various levels indicated that definite relative movement was present.

The tendency of the change centers fluctuated very slightly in layers above 4 kilometers from day to day except in instances where centers split. Finally, contributing density changes above 20 kilometers averaged 2 mbs.

In conclusion a statement should be made of the use of density change centers as an aid in the prognostication of constant pressure charts. As was noted previously, centers at specific layers were easily tracked at 90 percent of the wind speed on a trajectory basis. Also noted was the relation of specific centers in a layer to troughs and ridges observed in the contours of a constant pressure surface in

The first part of the paper is devoted to the study of the  
 properties of the function  $f(x) = \frac{1}{x}$  for  $x > 0$ . It is shown that  
 this function is strictly decreasing and convex. The second part  
 of the paper is devoted to the study of the function  $f(x) = \frac{1}{x^2}$  for  
 $x > 0$ . It is shown that this function is strictly decreasing and  
 convex. The third part of the paper is devoted to the study of the  
 function  $f(x) = \frac{1}{x^3}$  for  $x > 0$ . It is shown that this function is  
 strictly decreasing and convex. The fourth part of the paper is  
 devoted to the study of the function  $f(x) = \frac{1}{x^4}$  for  $x > 0$ . It is  
 shown that this function is strictly decreasing and convex. The  
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 $f(x) = \frac{1}{x^5}$  for  $x > 0$ . It is shown that this function is strictly  
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 the study of the function  $f(x) = \frac{1}{x^6}$  for  $x > 0$ . It is shown that  
 this function is strictly decreasing and convex. The seventh part  
 of the paper is devoted to the study of the function  $f(x) = \frac{1}{x^7}$  for  
 $x > 0$ . It is shown that this function is strictly decreasing and  
 convex. The eighth part of the paper is devoted to the study of  
 the function  $f(x) = \frac{1}{x^8}$  for  $x > 0$ . It is shown that this function  
 is strictly decreasing and convex. The ninth part of the paper is  
 devoted to the study of the function  $f(x) = \frac{1}{x^9}$  for  $x > 0$ . It is  
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 tenth part of the paper is devoted to the study of the function  
 $f(x) = \frac{1}{x^{10}}$  for  $x > 0$ . It is shown that this function is strictly  
 decreasing and convex.

that layer. During investigation of this relationship, several smoothed over short waves were noted at the 500 mb level and were added in dashed lines (See Plates I-V). It is believed that density change centers can be accurately prognosticated and do indicate in greater detail short wave features of contour charts. Mean density may be found for a layer by known differential methods. Taking the differential of these mean curves between two consecutive days will give mean density change centers for the layer. For a period of two weeks, the author attempted this procedure for the layer 1000 to 500 mbs. Results lacked the detail found by methods presented in this paper, but this is believed correctable to considerable extent by using a smaller layer such as 700 to 500 mbs. This technique is offered not as a complete method of prognostication in itself, but rather as an aid to methods presently used.



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1. The first of these is the fact that the population of the country is increasing rapidly, and this is a source of great danger to the country.
2. The second is the fact that the country is becoming more and more dependent on foreign trade, and this is a source of great danger to the country.
3. The third is the fact that the country is becoming more and more dependent on foreign capital, and this is a source of great danger to the country.
4. The fourth is the fact that the country is becoming more and more dependent on foreign technology, and this is a source of great danger to the country.
5. The fifth is the fact that the country is becoming more and more dependent on foreign labor, and this is a source of great danger to the country.
6. The sixth is the fact that the country is becoming more and more dependent on foreign food, and this is a source of great danger to the country.
7. The seventh is the fact that the country is becoming more and more dependent on foreign clothing, and this is a source of great danger to the country.
8. The eighth is the fact that the country is becoming more and more dependent on foreign medicine, and this is a source of great danger to the country.
9. The ninth is the fact that the country is becoming more and more dependent on foreign education, and this is a source of great danger to the country.
10. The tenth is the fact that the country is becoming more and more dependent on foreign culture, and this is a source of great danger to the country.



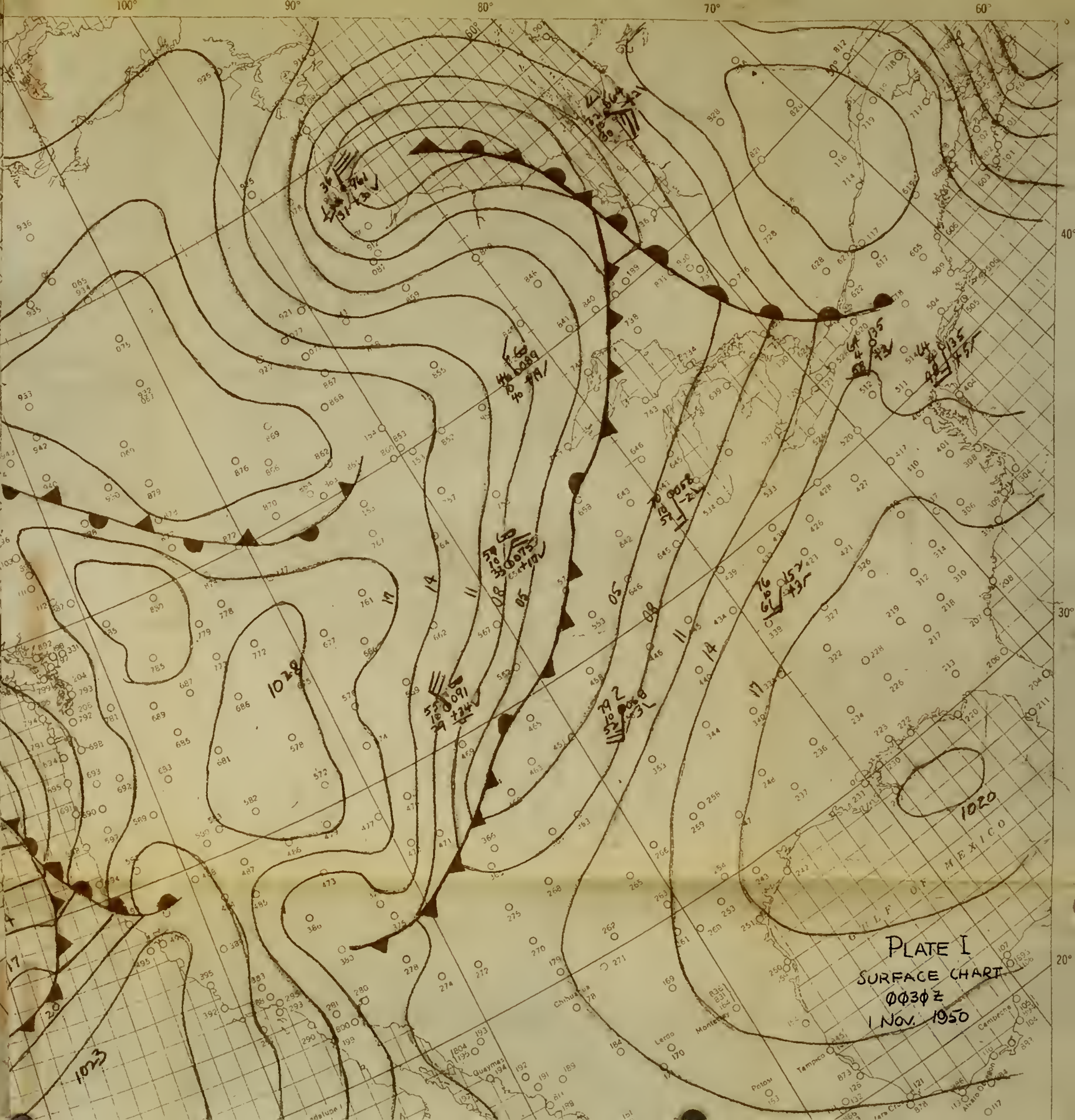
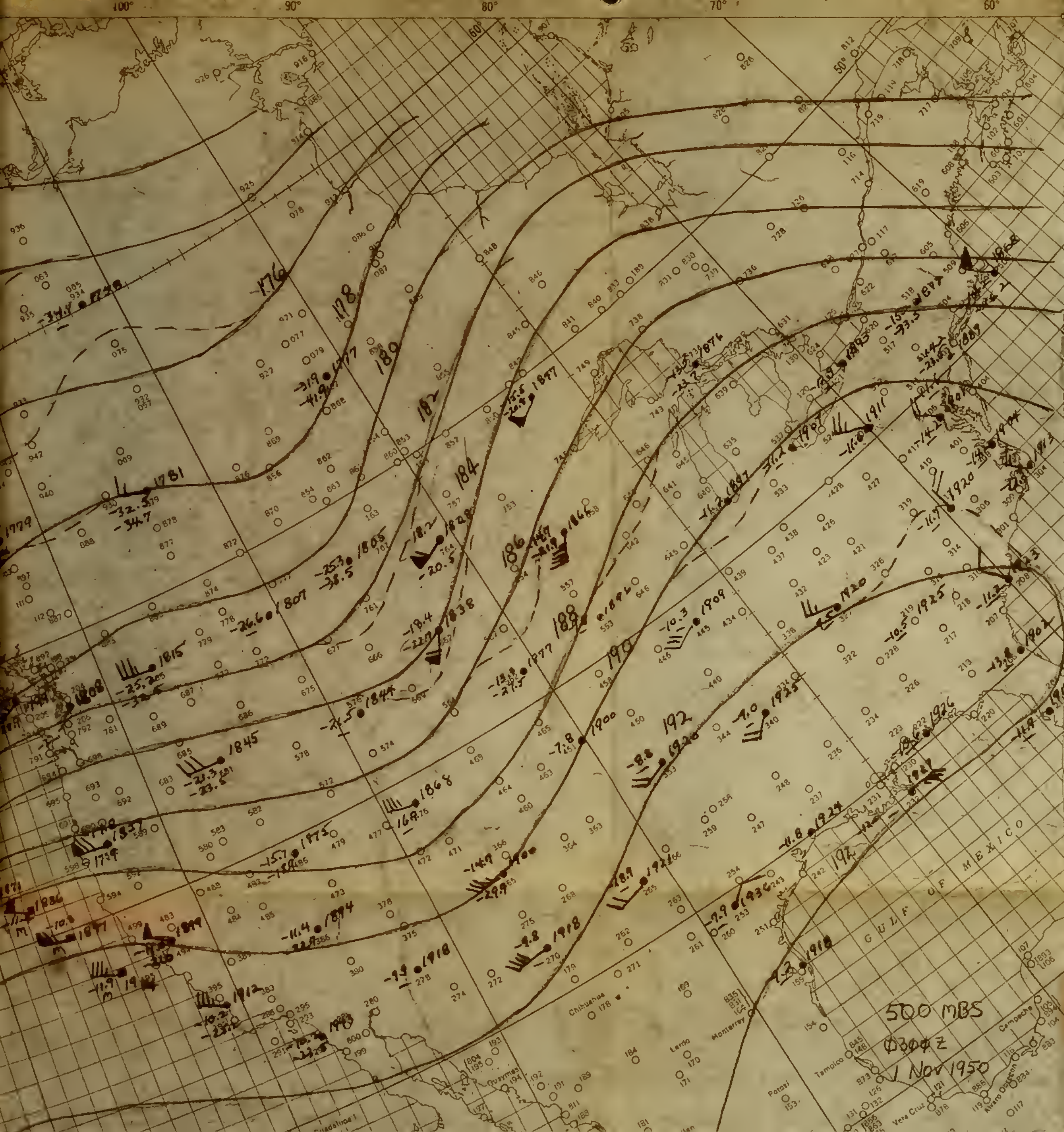
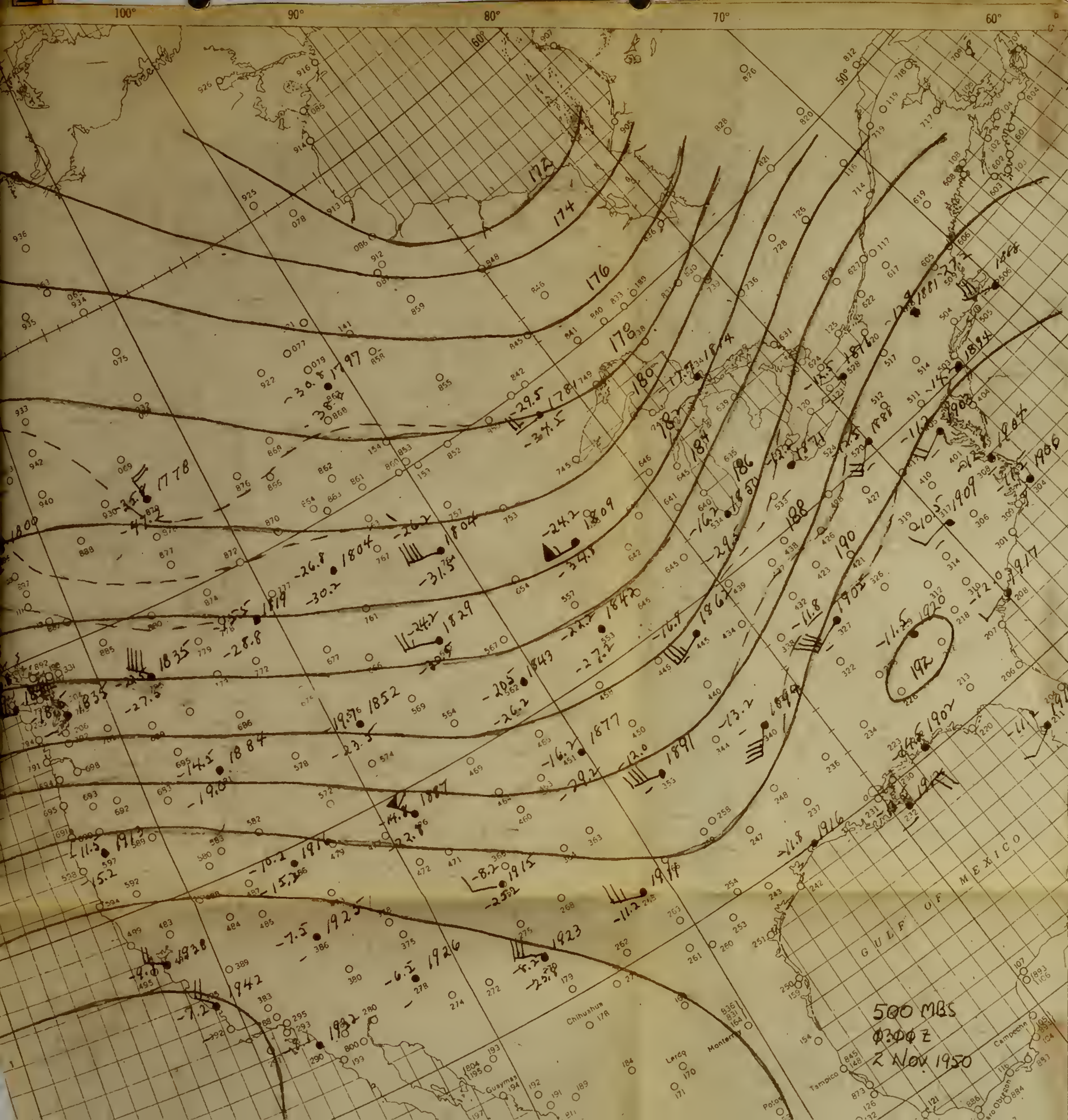


PLATE I  
SURFACE CHART  
0030Z  
1 Nov. 1950



1. The first of these is the fact that the number of persons who have been admitted to the office of Justice of the Peace has increased from 1860 to 1870.
2. The second is the fact that the number of persons who have been admitted to the office of Justice of the Peace has increased from 1860 to 1870.
3. The third is the fact that the number of persons who have been admitted to the office of Justice of the Peace has increased from 1860 to 1870.
4. The fourth is the fact that the number of persons who have been admitted to the office of Justice of the Peace has increased from 1860 to 1870.
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9. The ninth is the fact that the number of persons who have been admitted to the office of Justice of the Peace has increased from 1860 to 1870.
10. The tenth is the fact that the number of persons who have been admitted to the office of Justice of the Peace has increased from 1860 to 1870.

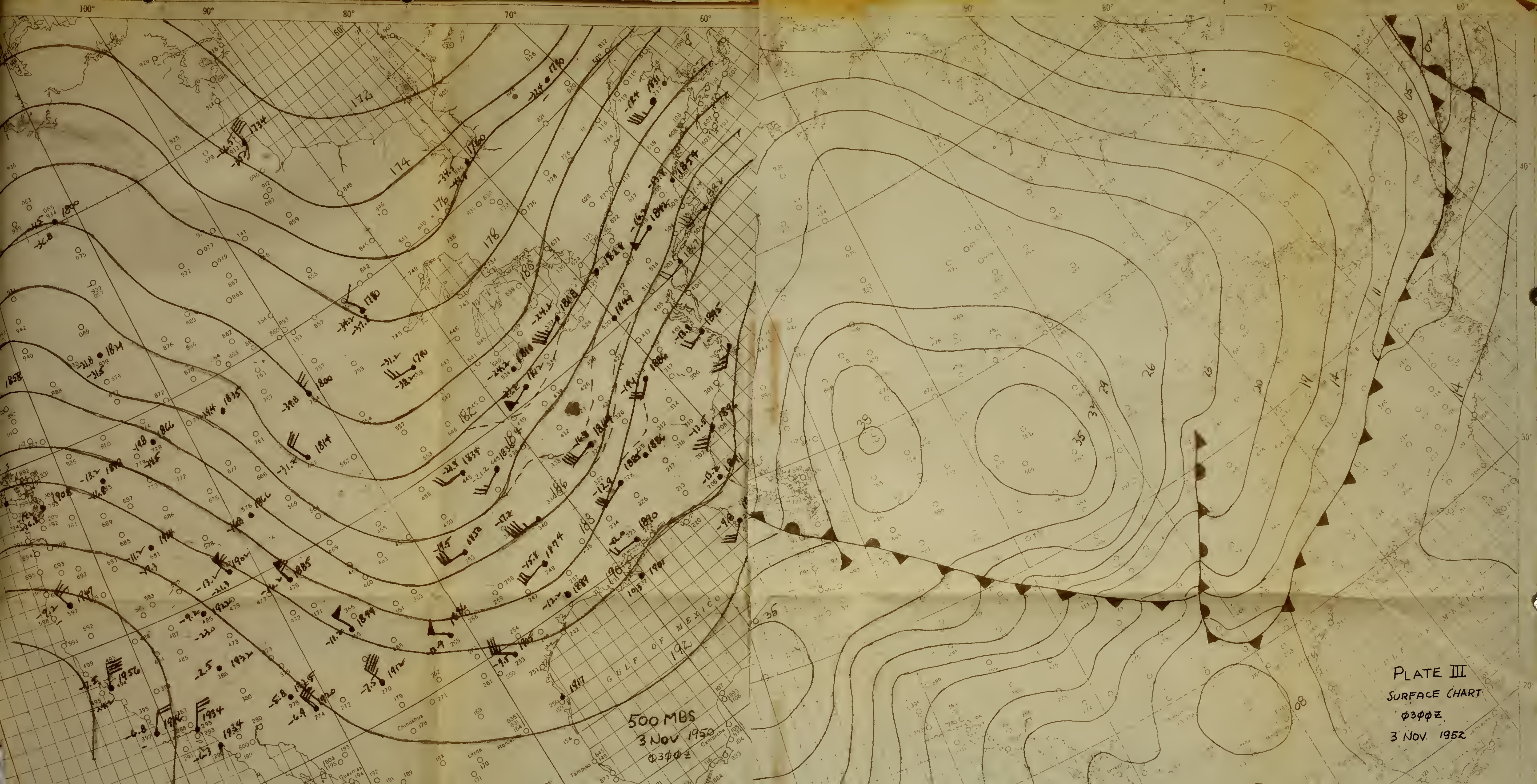




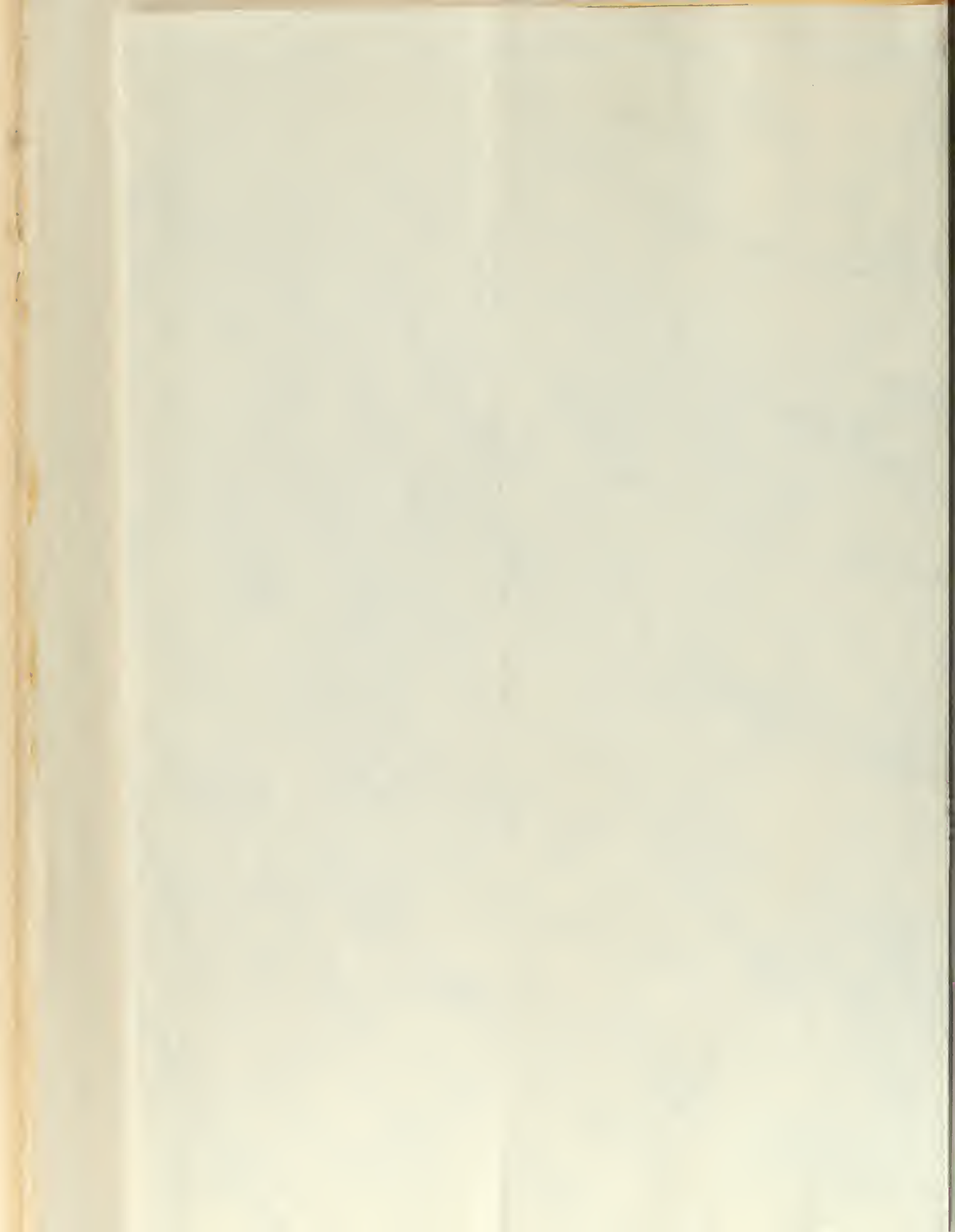




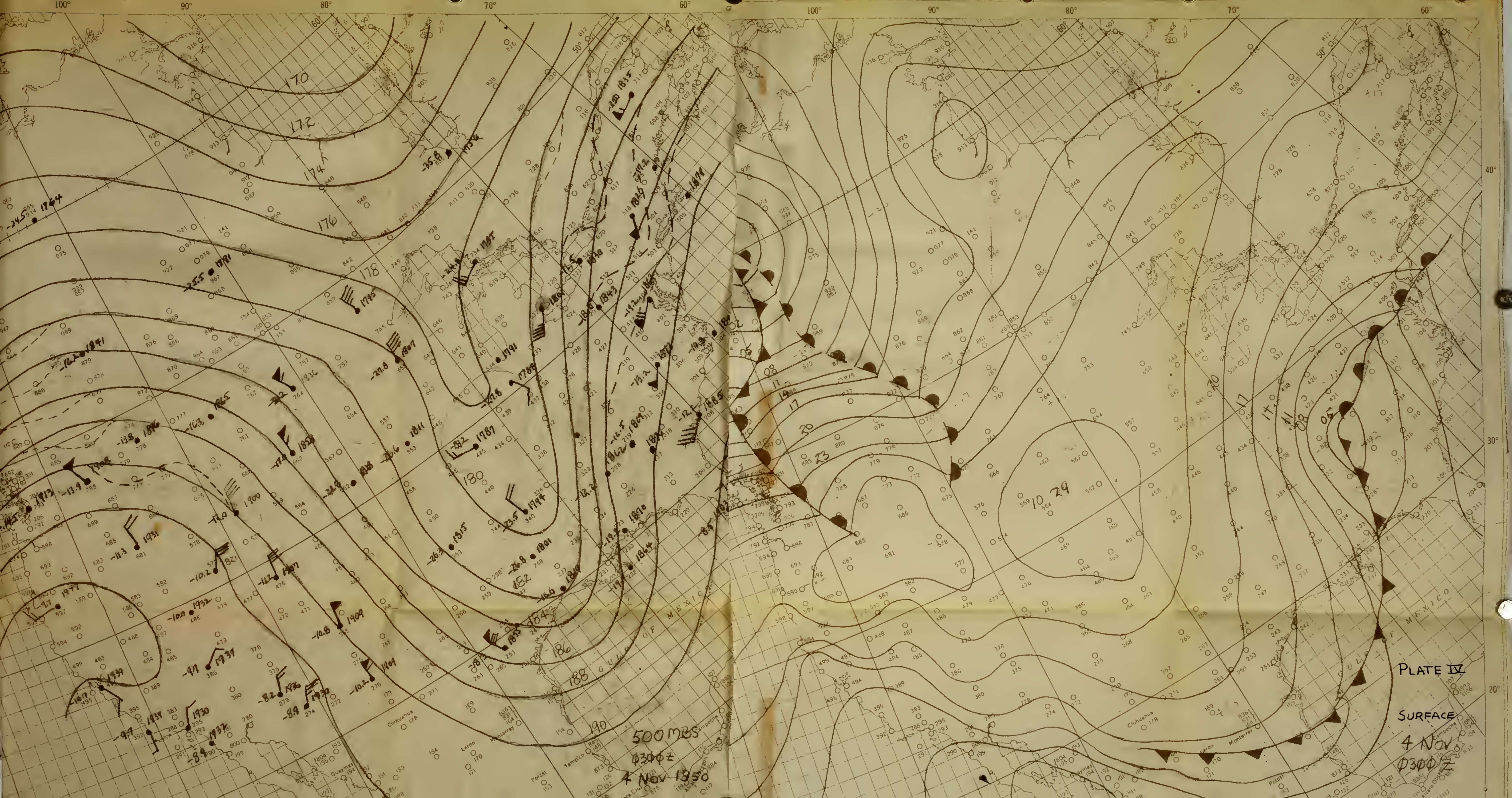








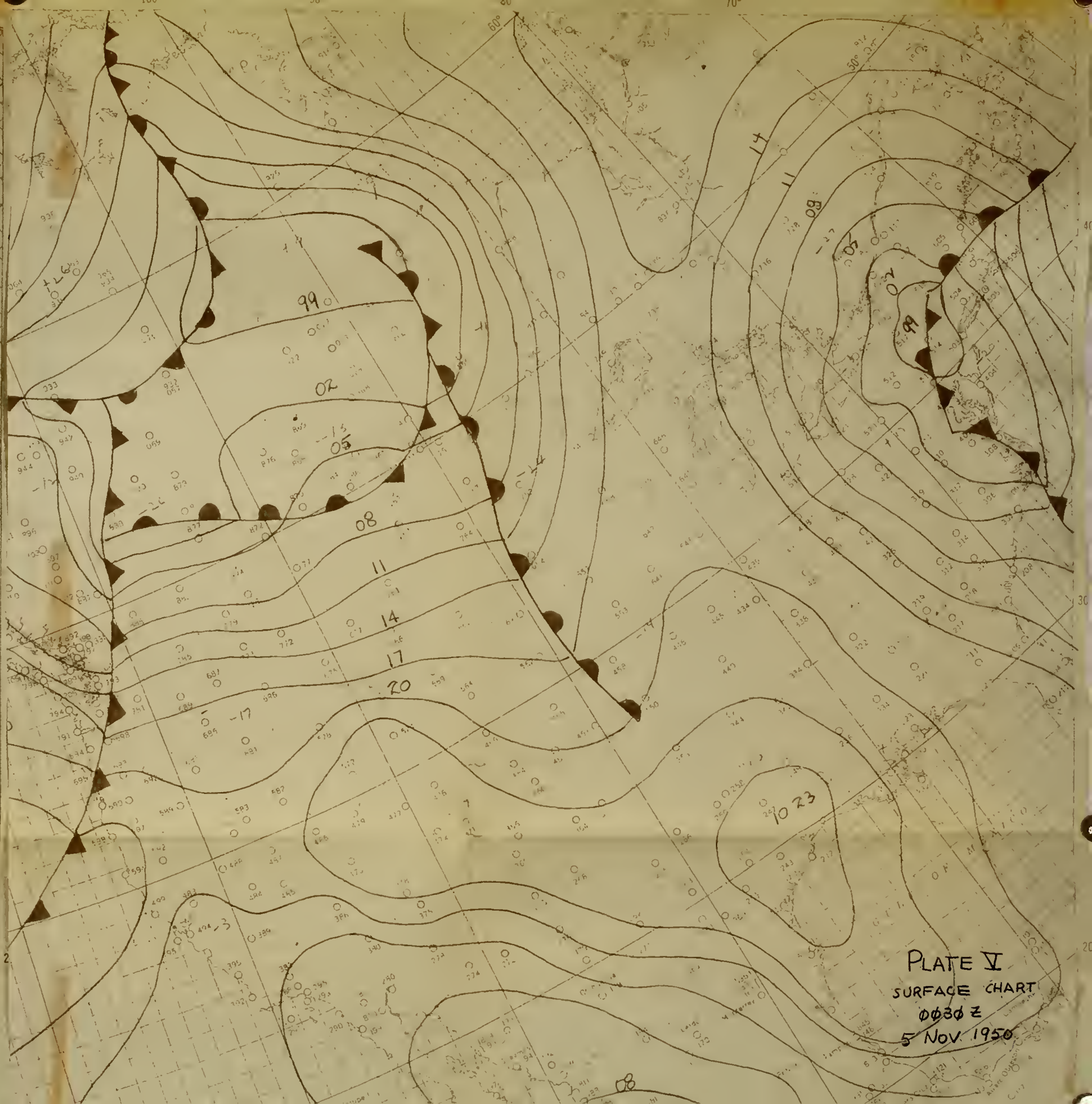
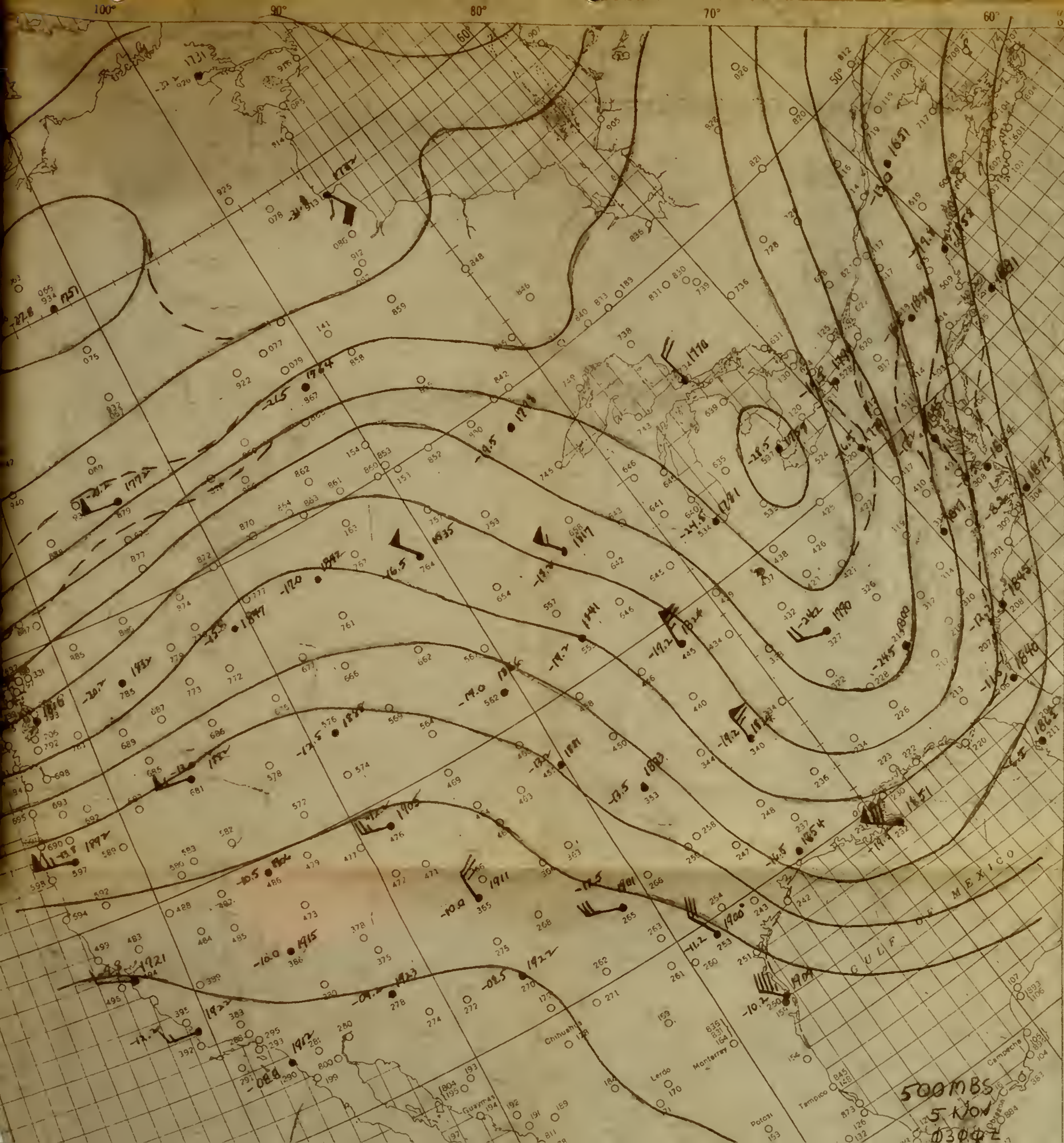








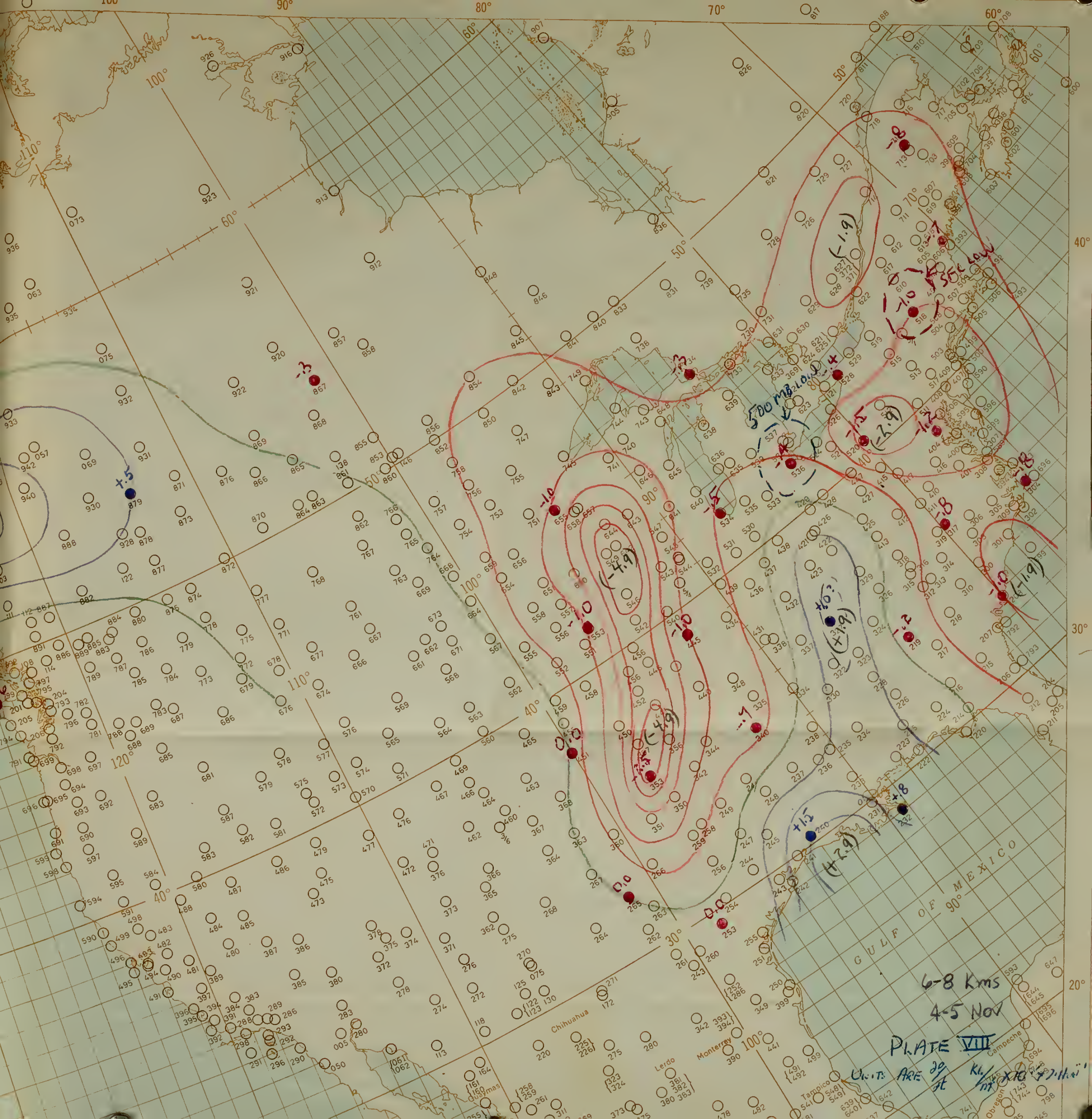












6-8 Kms  
4-5 Nov

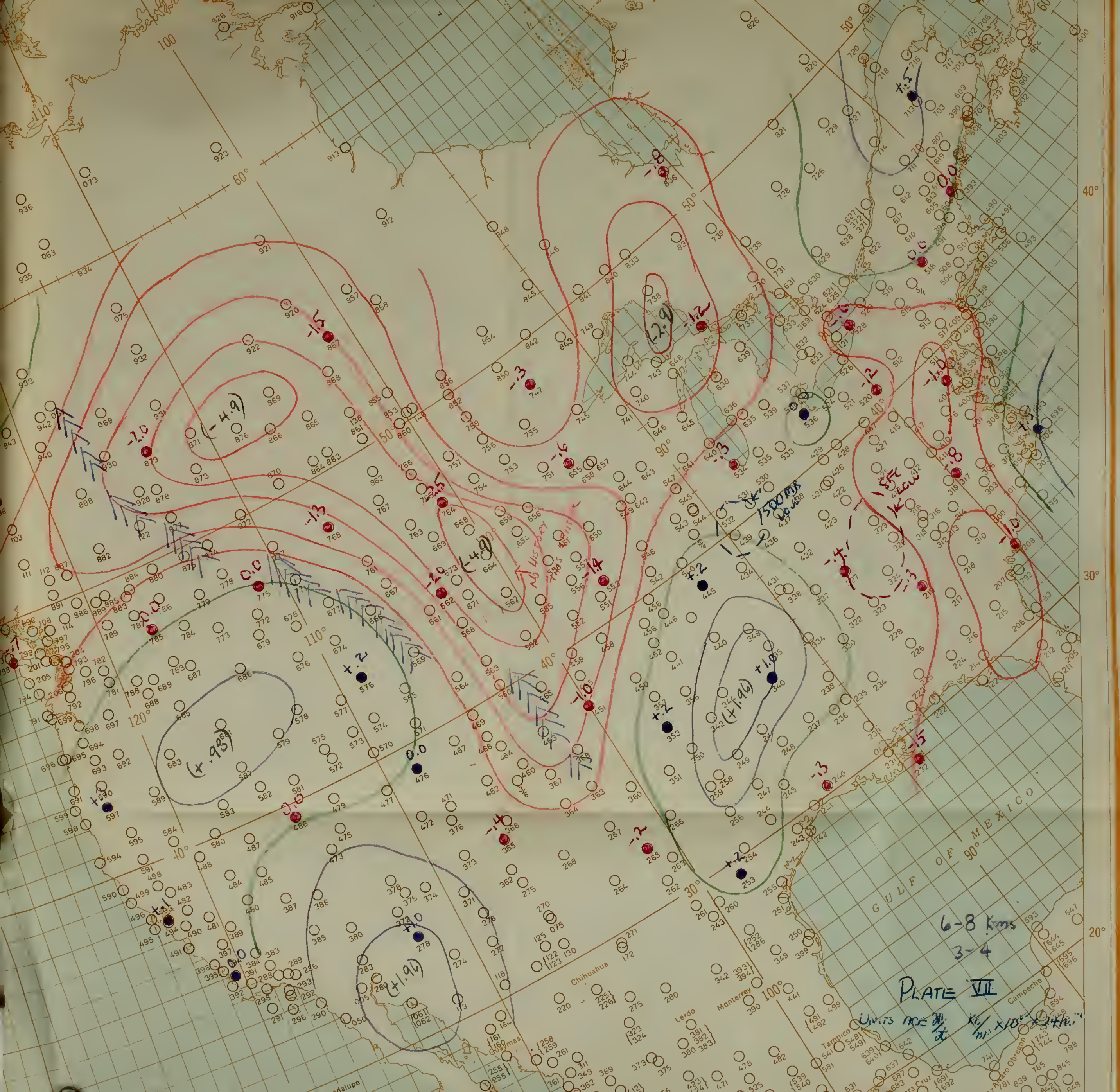
PLATE VIII

Units ARE 29/100  
K/1000 ft





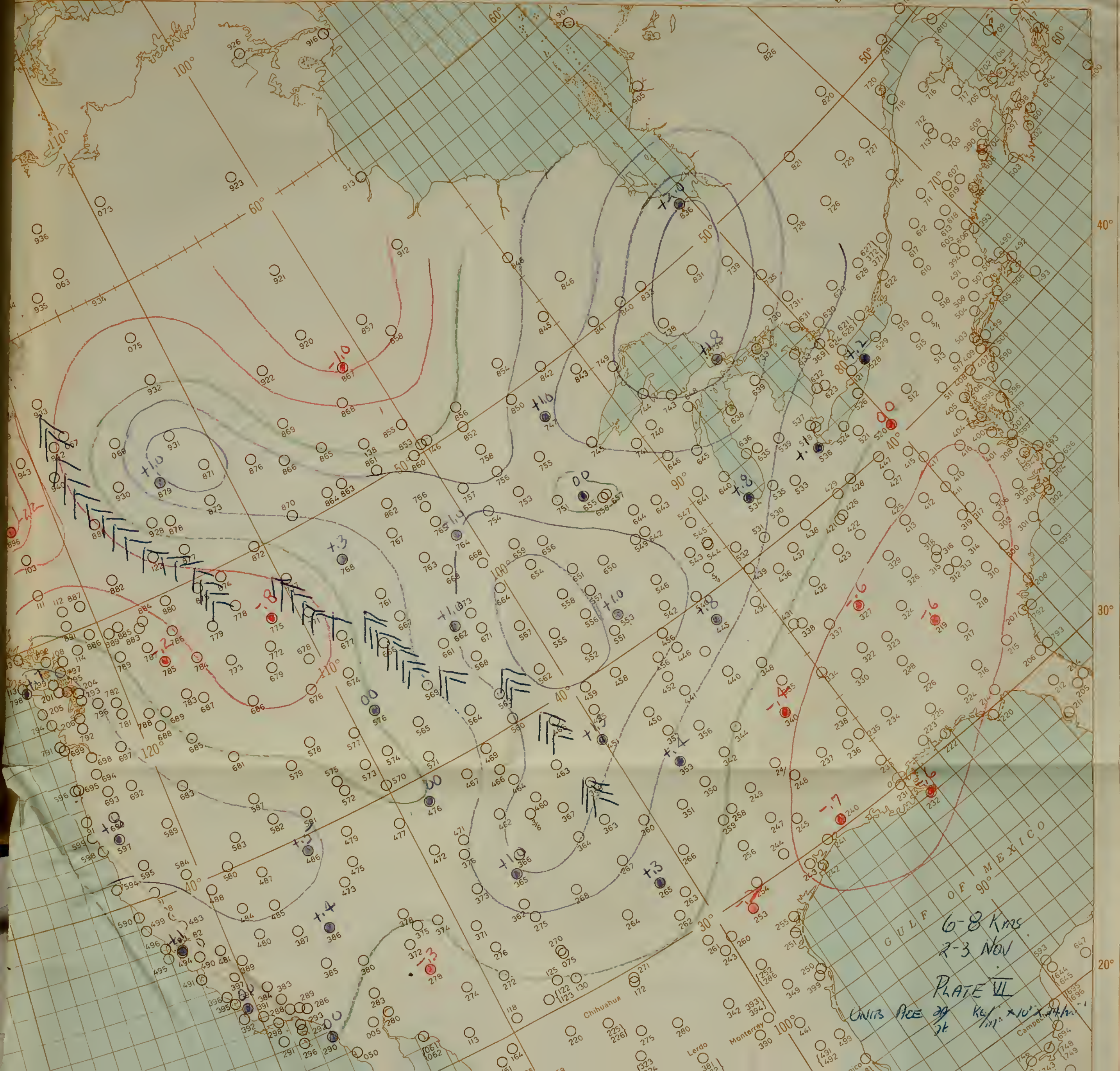


















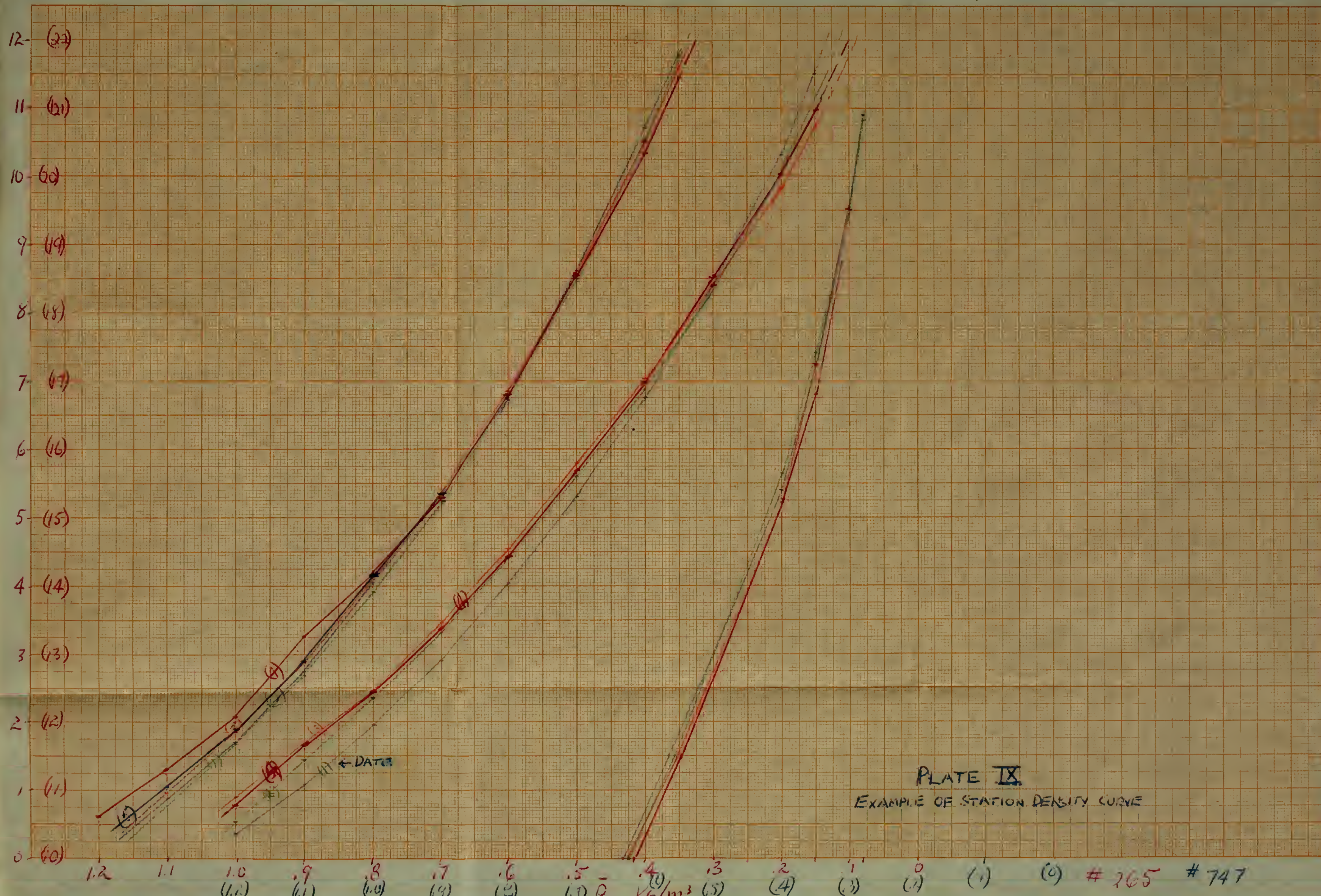


PLATE IX  
 EXAMPLE OF STATION DENSITY CURVE











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Westerlies.

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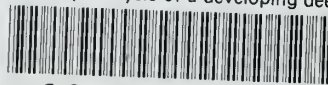
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